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The fate of massive F-strings

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Abstract: We calculate the semi-inclusive decay rate of an average string state with compactification both in the bosonic string theory and in the superstring theory. We also apply this calculation to a brane-inflation model in a warped geometry and find that the decay rate is greatly suppressed if final strings are all massive and enhanced with one final string massless, which may pose a challenge to this class of models.

Keywords: Superstrings and Heterotic Strings, Tachyon Condensation.

Contents

1. Introduction

The search for the imprints of the string theory in the cosmology is one of the most important problems in string cosmology. Cosmic string [1], as a promising candidate, has drawn much attention in the past few years. The renewed interest [3, 4] in the cosmic string resides in the fact that in the low-energy models the cosmic strings could be created after inflation. This is in contrast with the prediction of the old Planck-scale models, where the cosmic strings produced before the inflation are "blown away"[2]. It has been shown that some strings, including fundamental string and solitonic strings, are sufficiently stable and a few of them may survive and be observable in near future through gravitational wave detectors such as LIGO and LISA.

In the string-inspired models on inflation, there is a class of models based on the brane-antibrane annihilation. In such configurations, the tachyon field develop. As tachyon condense, very massive closed string states are created, which could be an origin of the cosmic string. However these massive fundamental string states are not stable and may decay to other massive or massless states. In order to understand the fate of a typical massive F-string state better, it is essential to study its decay systematically. On the other hand, even without considering its phenomenological implication, the study of the decay of a typical massive string state is an interesting issue on its own right in string theory. From the investigation of the rolling tachyon [5] and the geometric tachyon realization in the (NS, D)-system [6], it has been known that the tachyon condensation will radiate various kinds of closed string states [7, 8], even though there does not exist a well-understood treatment of the backreaction yet. To understand the tachyon condensation and the mysterious open/close string duality [9], the study of the fate of these massive string states is necessary. This is the question we will address in this note.

The decay of massive strings is an old problem of string theory and it is generally difficult to extract detailed knowledge from conventional calculations mainly due to the exponentially growing state density. States at the same mass level may be very different, especially their decay properties. For example, there exist some peculiar string states, often with large angular momentum, which are extraordinarily stable, with lifetime proportional to some positive power of their masses [13] [15].

Fortunately in a realistic situation, such as the rolling tachyon case, details of string states are not important, and we are only interested in some sort of averaged decay rates. A peculiar long-lived string state may be created but with much smaller probability, it must be in an extremal end of a distribution in which the typical string states are amply produced and more interesting. The lifetimes of the massive strings are cleverly computed in this spirit [16]. In [12] and [13], massless string emission from heavy strings were considered for both bosonic and superstrings, where the authors average over the initial states of the same mass level and fixed four momentum and sum over all possible states for the remnant string, a thermal spectrum was obtained. Manes considered in [14] the emission spectrum of the strings of all mass levels for bosonic (open and closed) strings. It was found there that the decay rates are universal, in the sense that the leading terms depend only on the mass level of the strings involved, and the more detailed properties of the states affect only the sub-leading terms.

To study the stringy properties relevant for cosmology, we need to consider the situation where some of the space-time dimensions are compactified. And in the realistic models building for early day inflation [11] and present day accelerated expansion [10], the superstrings rather than the bosonic strings are used. So in this note, we consider first the decay process of the bosonic strings with toroidal compactification in section 2. Here the KK modes and winding modes enter and make the process much more complicated. And in section 3, we go on to discuss the superstring process in the flat space where the universal expressions similar to [14] are found. We do compactification in section 4 to calculate the emission rate in a more realistic string inflation model. Section 5 includes our conclusions and discussions.

2. Bosonic string decay: with compactification

In this section, we study the emission spectrum of massive closed bosonic strings in the case where d_c out of the total D dimensions are compactified on a flat torus. A closed string state is characterized by its KK momentum numbers n_i, winding numbers w_i, and its mass M and momentum k in the noncompact dimensions. The mass shell condition reads

$$\alpha' M^2 = 4(N_R - 1) + \alpha' Q_+^2 = 4(N_L - 1) + \alpha' Q_-^2 \tag{2.1}$$

where $Q_\pm = \sum \frac{n_i}{R_i} \pm \frac{w_i R_i}{\alpha'}$, and $i = 1, \ldots, d_c$. For later convenience we also define

$$4M_{RL}^2 = M^2 - Q_\pm^2\,. \tag{2.2}$$

The decay amplitude for closed strings can be factorized into the left and the right parts, and each part can be calculated in a way similar to open strings. Consider first the string of given mass M, winding w_i and KK momentum n_i in its center of mass frame decays into two strings with mass M_1 and M_2, winding w_{1i} and w_{2i}, KK momentum n_{1i} and n_{2i}, with similar mass shell conditions

$$\alpha' M_1^2 = 4(N_{1R} - 1) + \alpha' Q_{1+}^2 = 4(N_{1L} - 1) + \alpha' Q_{1-}^2 \,, \tag{2.3}$$

and

$$\alpha' M_2^2 = 4(N_{2R} - 1) + \alpha' Q_{2+}^2 = 4(N_{2L} - 1) + \alpha' Q_{2-}^2 \,. \tag{2.4}$$

In the following, we set $\alpha' = \frac{1}{2}$. Energy conservation gives

$$M = \sqrt{M_1^2 + k^2} + \sqrt{M_2^2 + k^2} \,, \tag{2.5}$$

with k the momentum in the noncompact dimension.

What we want to consider is the averaged semi-inclusive two-body decay rate. That is, for the initial string, we average over all states of some given mass, winding and KK momentum. For one of the two final strings, we sum over all states with some given mass, winding and KK momentum; only the other string's state is fully specified (by keeping explicit its vertex operator).

This decay rate can be written as

$$\Gamma_{\mathrm{semi-incl}} = \frac{A_{D-d_c}}{M^2} g_c^2 \frac{F_L}{\mathcal{G}(N_L)} \frac{F_R}{\mathcal{G}(N_R)} k^{D-3-d_c} \prod_i^{d_c} R_i^{-1} \tag{2.6}$$

with closed string coupling g_c, compactification radius R_i, and numerical coefficient $A_p = \frac{2^{-p}\pi^{\frac{3-p}{2}}}{\Gamma(\frac{p-1}{2})}$, and F_L and F_R are given by

$$F_L = \sum_{\Phi_i|_{N_L}} \sum_{\Phi_f|_{N_{2L}}} |\langle \Phi_f | V_L(n_{1i}, w_{1i}, k) | \Phi_i \rangle|^2 \tag{2.7}$$

$$F_R = \sum_{\Phi_i|_{N_R}} \sum_{\Phi_f|_{N_{2R}}} |\langle \Phi_f | V_R(n_{1i}, w_{1i}, k) | \Phi_i \rangle|^2 \,. \tag{2.8}$$

The vertex operator for the one of the final state strings has been factorized into the left part V_L and the right part V_R, whose exponential parts look like $e^{ik_L \cdot X_L}, e^{ik_R \cdot X_R}$, with $-k_L^2 = M_1^2 - Q_{1-}^2$, which we define to be $4m_L^2$ for later comparison with open strings. In the flat spacetime the normalization of the initial state and the final state two-body phase space contribute a factor $\frac{k^{D-3}}{M^2} d\Omega$, while the integration over the solid angle Ω gives A_D. Now momenta in the d_c compact dimensions are discrete, so we convert the integration to a summation and this results in the factor $\prod_i^{d_c} R_i^{-1}$.

The sum in a fixed level is hard to perform, this can be done with a trick: we insert a projector P_N over states of level N, and then sum over states of all levels, in the end we obtain a trace [12]. Let

$$P_N = \oint_C \frac{dz}{z} z^{\hat{N} - N} \tag{2.9}$$

$$P_N |\Phi_{N'}\rangle = \delta_{NN'} |\Phi_{N'}\rangle \,, \tag{2.10}$$

so the amplitude squared can be written as

$$F_L = \oint_C \frac{dz}{z} z^{-N_L} \oint_{C'} \frac{dz'}{z'} z'^{-N_{2L}} \operatorname{Tr}[z^{\hat{N}} V_L^\dagger(k,1) z'^{\hat{N}} V_L(k,1)] \,, \tag{2.11}$$

where we place the vertex operator at $z = 1$, takeing advantage of the $\mathrm{SL}(2,\mathbb{R})$ invariance of the open string tree amplitude. In the following, we define $w = zz'$, and $v = z'$.

2.1 The recursion relation

Manes observed that [14] the trace in (2.11) describing a tree level process can be carried out by converting it to the two point correlation function on a cylinder with modulus t, where

$$it = -\frac{2\pi i}{\ln w} \,. \tag{2.12}$$

Note that [17]

$$\int d^D p \operatorname{Tr}[z^{L_0} V^\dagger(k,1) z'^{L_0} V(k,1)] = f(w)^{2-D} \left(\frac{-2\pi}{\ln w}\right)^{D/2} \langle V^\dagger(k,1)\ V(k,v)\rangle \,, \tag{2.13}$$

where

$$f(w) = \prod_{n=1}^{\infty} (1 - w^n)^{2-D} \,. \tag{2.14}$$

The second factor comes from the momentum integral over the zero modes, which does not appear in the definition (2.7) and (2.8) of F_{LR}. Remember that compactification does not affect the oscillation modes, so we have

$$\operatorname{Tr}[z^{\hat{N}} V_L^\dagger(k_L,1) z'^{\hat{N}} V(k_L,1)] = f(w)^{2-D} \langle V'^\dagger_L(k_L,1) V'_L(k_L,v)\rangle \,, \tag{2.15}$$

where the prime means that $\partial_\tau^n X^\mu(v)$ is replaced by $v^{\hat{N}} \partial_\tau^n X^\mu(1) v^{-\hat{N}}$ in the vertex operator so that zero modes are excluded, and $D = 26$ is the full space-time dimension. Thus the $master formula$ for emission of general states including KK modes and winding is

$$F_L = \oint \frac{dw}{w} w^{-N_L} f(w)^{2-D} \oint \frac{dv}{v} v^{N_L - N_{2L}} \langle V'^\dagger_L(k_L,1) V'(k_L,v)\rangle \,. \tag{2.16}$$

It can be rewritten as

$$F_L = \oint_{C_w} \frac{dw}{w} w^{-N_L} f(w)^{2-D} \mathcal{I}_{N_L - N_{2L}}(w) \tag{2.17}$$

$$\mathcal{I}_n = \oint_{C_v} \frac{dv}{v} v^n \langle V'^\dagger_L(k_L,1) V'_L(k_L,v)\rangle \tag{2.18}$$

where the contours satisfy $|w| < |v| < 1$, for $|z|, |z'| < 1$. A general vertex operator $V(k,z)$ is got from a physical state

$$|\Phi\rangle = \sum c_{\mu_1 \ldots \mu_m} \alpha^{\mu_1}_{-n_1} \ldots \alpha^{\mu_m}_{-n_m} |0;k\rangle \tag{2.19}$$

by replacing α^μ_{-n} by $\frac{i\partial^n_\tau X^\mu(z)}{(n-1)!}$, and $|0;k>$ by $e^{ikX(z)}$, and the normal ordering is assumed. We take the gauge[1]

$$k^{\mu_1}c_{\mu_1\mu_2\ldots\mu_m} = k^{\mu_2}c_{\mu_1\mu_2\ldots\mu_m} = \cdots = k^{\mu_m}c_{\mu_1\mu_2\ldots\mu_m} = 0\,. \tag{2.20}$$

for which the contractions of exponentials with ∂X like terms vanish. Then the correlator in $\mathcal{I}_n(w)$ factorizes into two parts, the contraction of two exponentials and the contractions without exponentials.

Remember that X scalar correlator on the cylinder is not changed by compactification, so

$$\langle e^{-ik_L X_L(1)}e^{ik_L X_L(v)}\rangle = e^{-\frac{1}{4}k_L^2\ln\hat\psi(v,w)} = \hat\psi(v,w)^{m_L^2}\,, \tag{2.21}$$

$$\langle\partial^n_\tau X^\mu_L(1)\partial^m_\tau X^\nu_L(v)\rangle = -(-)^n\eta^{\mu\nu}(v\partial_v)^{n+m}\ln\hat\psi(v,w)\,, \tag{2.22}$$

with

$$\hat\psi(v,w) = (1-v)\prod_{n=1}^\infty\frac{(1-w^n v)(1-w^n/v)}{(1-w^n)^2}\,. \tag{2.23}$$

In the above correlators, substitution has been made to account for the prime in the vertex operator. And ∂_τ is $v\partial_v$ for euclidean proper time $\tau=\ln z$. The correlator in $\mathcal{I}_n$ can thus be written as

$$\langle {V'}^\dagger_L(k_L,1)V'_L(k_L,v)\rangle = \hat\psi^{m_L^2}\mathcal{P}(\Omega,\partial_\tau\Omega\ldots)\,, \tag{2.24}$$

with

$$\Omega(v,w) = \langle\partial_\tau X_L(1)\partial_\tau X_L(v)\rangle = \sum_{n=1}^\infty n\left(v^n+\frac{w^n(v^n+v^{-n})}{1-w^n}\right)\,. \tag{2.25}$$

The contour integration in $\mathcal{I}_n$ was computed in [14]. Since the similar calculation is to be performed in the superstring case, we in the following repeat the steps of [14]. Note that

$$\hat\psi(w^{-1}v,w) = -\frac{v}{w}\hat\psi(v,w)\,, \tag{2.26}$$

$$\partial^n_\tau\Omega(w^{-1}v,w) = \partial^n_\tau\Omega(v,w)\,. \tag{2.27}$$

As $m_L^2 = 2(N_L-1)$ is even, this makes the minus sign in (2.26) irrelevant. The integration on the new contour C'_v carries out to be

$$\oint_{C'_v}\frac{dv'}{v'}v'^n\langle {V'}^\dagger_L(k_L,1)V'_L(k_L,v')\rangle = w^{-n-m_L^2}\oint_{C_v}\frac{dv}{v}v^{n+m_L^2}\langle {V'}^\dagger_L(k_L,1)V'_L(k_L,v)\rangle\,, \tag{2.28}$$

which is just

$$w^{-n-m_L^2}\mathcal{I}_{n+m_L^2}(w)\,. \tag{2.29}$$

The difference between the two contours is the integration over a small contour around the singularity $v=1$ or $\tau=\ln v=0$. The function $\hat\psi(v,w)$ has the behavior [14]

$$\hat\psi(v,w) = -\tau\left(1+\frac{1}{2}\tau\right)+O(\tau^3) \tag{2.30}$$

[1]Strictly speaking, such a choice of gauge only exist in the critical dimension: for bosonic string $D=26$ and for superstring $D=10$, which are implied in this paper.

near $v = 1$. So near $v = 1$, the correlators have the following asymptotic behavior

$$\langle e^{-ik_L X_L(1)} e^{-ik_L X_L(v)} \rangle = \hat{\psi}(v,w)^{m_L^2} = \tau^{m_L^2}\left(1 + \frac{1}{2}m_L^2\tau + O(\tau^2)\right), \tag{2.31}$$

$$\langle \partial_\tau^n X_L(1) \partial_\tau^m X_L(v) \rangle = -\partial_\tau^{n+m-2}\Omega(v,w) = -\frac{1}{\tau^{n+m}} + O\left(\frac{1}{\tau^{n+m-2}}\right). \tag{2.32}$$

Note that one more partial derivative for each vertex operator gives an additional τ^{-2} in (2.32), but at the same time, the corresponding state is more excited, contributing τ^2 to (2.31). The two factors exactly cancel

$$\langle V'^\dagger(k,1) V'(k,v) \rangle = \frac{c}{\tau^2}(1 + \frac{1}{2}m_L^2\tau) + O(1), \tag{2.33}$$

where the coefficient can be fixed to $c = 1$ by state normalization. So the contour integration around $v = 1$ in our case with compactification is the same as in the noncompact case

$$\oint_C \frac{dv}{v} v^n \langle V'^\dagger_L(k_L,1) V'_L(k_L,v) \rangle = n + \frac{m_L^2}{2}. \tag{2.34}$$

We get the following recursion relation

$$\mathcal{I}_{n+m_L^2}(w) = w^{n+m_L^2}\left[\mathcal{I}_n(w) + n + \frac{m_L^2}{2}\right], \tag{2.35}$$

which, used in sequence, can be recast into a series summation

$$\mathcal{I}_n(w) = \sum_{p=1}^{A}\left[n - m_L^2\left(p - \frac{1}{2}\right)\right] w^{np - \frac{1}{2}m_L^2(p^2-p)} + \mathcal{I}_\nu(w) w^{\nu A + \frac{1}{2}m_L^2(A^2+A)}, \tag{2.36}$$

where $\nu = n - m_L^2 A$, with A an integer. Then the contour integration in (2.16) can be written as

$$F = \sum_{p=1}^{A}\left(n - m_L^2\left(p - \frac{1}{2}\right)\right)\mathcal{G}[N_L - np + \frac{1}{2}m_L^2(p^2-p)] + F_{NU}, \tag{2.37}$$

where $n = N_L - N_{2L}$, and

$$F_{NU} = \oint_{C_w} \frac{dw}{w} w^{-N} f(w)^{2-D} \mathcal{I}_\nu(w) w^{\nu A + \frac{1}{2}m_L^2(A^2+A)}, \tag{2.38}$$

and the generating function for the mass level density

$$\mathrm{Tr}\, w^{\hat{N}} = f(w)^{2-D} = \sum_{N=0}^{\infty} \mathcal{G}(N) w^N. \tag{2.39}$$

has been used.

The eq. (2.37) is useful in that all terms except the last one in the decreasing series do not depend on the details of any of the three string states involved (two are natural from the definition eq. (2.7)), thus knowing their levels is enough to determine them fully. They are hence universal. The last term F_{NU} is non-universal. The recursion relation tells that the independent functions are $\mathcal{I}_0, \ldots, \mathcal{I}_{m_L^2/2}$, which depends on the particular vertex operators we inserted and so are not universal. The contribution from F_{NU} to F is generically negligible, in comparison with the universal part contribution, except the case $A = 0$ which happens when the emitted states carry a large fraction of the total mass.

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2.2 The decay rate

In the following, we shall calculate the decay rate (2.6). In the above discussions, we have fixed the levels of the incoming string states and one of the outgoing string states, which are $N_{L,R}, N_{2L,2R}$ respectively. From the mass-shell conditions, we know that once we fix the quanta of the incoming string states, the outgoing string states could have various kinds of masses, KK-momenta and windings, with respect to the energy condition (2.5), and conservations:

$$Q_- = Q_{1-} + Q_{2-}\,, \qquad Q_+ = Q_{1+} + Q_{2+}\,. \tag{2.40}$$

One important observation is that we have inequality

$$\sqrt{N_{1L}} + \sqrt{N_{2L}} \leq \sqrt{N_L}\,. \tag{2.41}$$

The equality saturate when

$$k = 0\,, \qquad \frac{M}{Q_-} = \frac{M_2}{Q_{2-}}\,, \tag{2.42}$$

where k is the momentum in the noncompact directions and M_2, Q_{2-} are the quanta of the outgoing string states with fixed level N_{2L}. The same inequality holds in the right-mover.

Given a very massive initial string of high level, its state density has the asymptotic form

$$\mathcal{G}(N) \sim N^{-\frac{D+1}{4}} e^{a\sqrt{N}}\,, \qquad a = 2\pi\sqrt{\frac{D-2}{6}}\,. \tag{2.43}$$

The ratio between the first two terms in (2.37) can be estimated to be

$$\frac{\mathcal{G}(N_L - n)}{\mathcal{G}(N_L - 2n + m_L^2)} \sim e^{a\left(\sqrt{N_L - n} - \sqrt{N_L - 2n + m_L^2}\right)}\,. \tag{2.44}$$

Using the inequality(2.41), it is at most of order

$$\exp\left(a(\sqrt{N_L} - \sqrt{N_{2L}})\right), \qquad \sqrt{N_{2L}} \geq \frac{\sqrt{N_L}}{2} \tag{2.45}$$

or

$$\exp(a(3\sqrt{N_{2L}} - \sqrt{N_L}))\,, \qquad \sqrt{N_{2L}} \leq \frac{\sqrt{N_L}}{2}\,. \tag{2.46}$$

In the extremal case $N_{2L} = N_L$, one can try to calculate $\mathcal{I}$ directly. Thus if generically $N_L > N_{2L} > \frac{N_L}{3}$, the first term dominates the whole summation in (2.37), and the other terms will be neglected to get

$$F_L \approx \left(N_L - N_{2L} - \frac{1}{2}m_L^2\right)\mathcal{G}(N_{2L})\,. \tag{2.47}$$

F_R can be carried out in the same way.

Note that in our approximation, $F_{L,R}$ do not depend on the details of the state specified in eq. (2.7) and (2.8) by the vertex operators, and all states of the same level are emitted with the same probability. Taking advantage of this, we can get the total decay rate for decays into arbitrary states of given mass, winding and KK momentum by simply multiplying eq. (2.6) by the state density $\mathcal{G}(N_1)$

$$\Gamma[(M,n_i,w_i)\to(m,n_{1i},w_{1i})+(M_2,n_{2i},w_{2i})]\approx A_{D-d_c}\frac{g_c^2}{M^2}\mathcal{N}_L\mathcal{N}_R\mathcal{G}_L\mathcal{G}_R k^{D-3-d_c}\prod_i^{d_c}R_i^{-1}\,, \tag{2.48}$$

where

$$\mathcal{N}_L=N_L-N_{2L}-\frac{1}{2}m_L^2\,,\qquad \mathcal{N}_R=N_R-N_{2R}-\frac{1}{2}m_R^2\,, \tag{2.49}$$

and

$$\mathcal{G}_L=\frac{\mathcal{G}(N_{1L})\mathcal{G}(N_{2L})}{\mathcal{G}(N_L)}\,,\qquad \mathcal{G}_R=\frac{\mathcal{G}(N_{1R})\mathcal{G}(N_{2R})}{\mathcal{G}(N_R)}\,. \tag{2.50}$$

Remember that

$$m_L^2=\frac{1}{4}(m^2-Q_{1-}^2)\approx 2N_{1L}\,. \tag{2.51}$$

As long as $N_{1L}\gg 1$ and $N_{2L}\gg 1$, we can use eq. (2.43) to write

$$\mathcal{G}_L\sim(2\pi T_H)^{-\frac{D-1}{2}}\left(\frac{N_{1L}N_{2L}}{N_L}\right)^{-\frac{D+1}{4}}e^{-\sqrt{2}t_L/T_H}\,, \tag{2.52}$$

with the Hagedorn temperature $T_H=\frac{1}{\pi}\sqrt{\frac{3}{D-2}}$ and $t_L=\sqrt{N_L}-\sqrt{N_{1L}}-\sqrt{N_{2L}}$ in a sense coming from the kinetic energy released in the decay process. We have in the above restored the multiplicative constant in front of the state density. Note that for later convenience, in our notation we set $\mathcal{G}(M)dN=\mathcal{G}(N)dN$, a little different from usual sense $\mathcal{G}(M)dM=\mathcal{G}(N)dN$.

From the discussion on the inequality (2.41), we know that the dominant decay channel is the original incoming string break into two strings with the same

$$\alpha=\frac{M}{Q_-}=\frac{m}{Q_{1-}}=\frac{M_2}{Q_{2-}} \tag{2.53}$$

and without releasing any kinetic energy $k=0$. In other words, the outgoing string states incline to carry proportional KK-momenta and windings to their masses. Furthermore, in such string breaking without releasing the kinetic energy, it is least probable that the original string break into two strings with the same mass and quanta.

To see more clearly the decay process, we may take into account the dependence on the momentum k and other quanta. An efficient way is to expand around the maximum of the exponential. It turns out to be

$$\mathcal{G}_L\sim\exp\left(-\frac{\sqrt{2}k^2}{16T_H}\sqrt{\frac{N_L}{N_{2L}}}\frac{1}{\sqrt{N_L}-\sqrt{N_{2L}}}\right)\exp\left(\frac{-\sqrt{2N_LN_{2L}}\epsilon^2}{(\alpha^2-1)^2(\sqrt{N_L}-\sqrt{N_{2L}})2T_H}\right) \tag{2.54}$$

where ϵ characterize the deviation of $\frac{M_2}{Q_{2-}}$ from α. Thus the dependence on k is still a gaussian distribution, which could be reduced to the one in [14] without compactification. The influence of the KK-momenta and windings is the change of the coefficients in the exponential

$$\exp\left(-\frac{\sqrt{2}k^2}{16T_H}\sqrt{\frac{N_L}{N_{2L}}}\frac{1}{\sqrt{N_L}-\sqrt{N_{2L}}}\right)=\exp\left(-\frac{\alpha}{\sqrt{\alpha^2-1}}\frac{k^2}{2T_H}\frac{M}{2mM_2}\right) \tag{2.55}$$

which tell us when α is huge, namely the original windings and KK-momentum is small, the distribution reduces to the one without compactification. Moreover, the contribution of KK-momentum and windings has also a gaussian distribution, and the total contribution could roughly be integrated out and give us a finite number. However, things are more interesting here: from the distribution of the ϵ, it seems that when $\alpha \to \infty$, which happens at $Q_- = 0$, all kinds of quanta of Q_{2-} could give the same contribution. This is just an illusion, due to the subtlety on the expansion. In this case, the outgoing strings tends to carry very small Q_{2-} quanta, with a distribution as $\exp(-\frac{MM_2}{m}\frac{\delta^2}{2T_H})$, where δ characterize the deviation from the extreme. In any case, one may take into account of all the contributions of the KK-momenta and windings well approximately by integrating over the distributions above.

With the above assumptions, eq. (2.49) is approximately

$$N_L - N_{2L} - \frac{1}{2}m_L^2 \approx \frac{\alpha^2-1}{\alpha^2}\frac{mM_2}{4}\,, \qquad N_R - N_{2R} - \frac{1}{2}m_R^2 \approx \frac{\beta^2-1}{\beta^2}\frac{mM_2}{4}\,, \tag{2.56}$$

where $\beta = M/Q_+$.

Now gathering all above together, we can calculate the total rate for a string with mass M, KK momentum n_i, winding w_i to produce a string with mass m, by integrating over noncompact momentum k (or say it in another way, summing over the other final string's mass level), and summing over partitions (n_{1i}, n_{2i}) and (w_{1i}, w_{2i}) with constraints $n_i = n_{1i} + n_{2i}$ and $w_i = w_{1i} + w_{2i}$.

First, we do the summations

$$\Gamma_m(M, n_i, w_i) = \sum_{N_2}\sum_{(n_{1i},n_{2i})}\sum_{(w_{1i},w_{2i})} \Gamma[(M, n_i, w_i) \to (m, n_{1i}, w_{1i}) + (M_2, n_{2i}, w_{2i})]\,. \tag{2.57}$$

To illustrate the picture more clearly, let us consider a special case, where the initial string carry very small Q_-. In this case, $\mathcal{G}_L\mathcal{G}_R$ can be divided into three parts: the kinetic energy, the KK modes and winding modes

$$\mathcal{G}_L\mathcal{G}_R \sim \mathcal{G}^k\mathcal{G}^{KK}\mathcal{G}^w\,, \tag{2.58}$$

where

$$\mathcal{G}^k = \exp\left[-\frac{k^2}{2T_H}\frac{M}{mM_2}\right], \tag{2.59}$$

$$\mathcal{G}^{KK} = \exp\left[-\frac{1}{2T_H}\sum_{i=1}^{d_c}\frac{1}{R_i^2}\left(\frac{n_{1i}^2}{m} + \frac{n_{2i}^2}{M_2} - \frac{n_i^2}{M}\right)\right], \tag{2.60}$$

$$\mathcal{G}^w = \exp\left[-\frac{2}{T_H}\sum_{i=1}^{d_c}R_i^2\left(\frac{w_{1i}^2}{m} + \frac{w_{2i}^2}{M_2} - \frac{w_i^2}{M}\right)\right]. \tag{2.61}$$

We see from eq. (2.59) that the probability to emit large kinetic energy strings is exponentially suppressed as expected. The factor (2.60) from the KK modes, when we fix $n_i = n_{1i} + n_{2i}$, has a saddle point which means that the processes satisfying $n_{1i}/m =$

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n_{2i}/M_2 are much preferred. Similarly the saddle point for the winding modes occurs at $w_{1i}/m = w_{2i}/M_2$. That is, for the final two strings, the heavier one tends to carry larger KK momentum and more windings. This is consistent with our general argument above. The decay rate can correspondingly be factorized into three parts

$$\Gamma_m(M, n_i, w_i) = \Gamma^k \sum_{(n_{1i}, n_{2i})} \mathcal{G}^{KK} \sum_{(w_{1i}, w_{2i})} \mathcal{G}^w \,, \tag{2.62}$$

where we make saddle point approximation for the noncompact momentum, that is we let $k = 0$, or $M_2 = M - m$, when considering the KK modes and winding modes. Note that the basic ϑ function has the expansion

$$\vartheta(\nu|\tau) \sum_{n=-\infty}^{\infty} \exp(\pi i n^2 \tau + 2\pi i n \nu) \,, \tag{2.63}$$

so the two summations in (2.62) which are actually summations over single argument $\sum_{n_{1i}=-\infty}^{\infty}$ and $\sum_{w_{1i}=-\infty}^{\infty}$, can be written more concretely in terms of ϑ functions

$$\sum_{(n_{1i}, n_{2i})} \mathcal{G}^{KK} \approx \prod_{i=1}^{d_c} e^{\beta_i} \vartheta(\nu_i|\tau_i) \,, \qquad \sum_{(w_{1i}, w_{2i})} \mathcal{G}^w \approx \prod_{i=1}^{d_c} e^{\beta'_i} \vartheta(\nu'_i|\tau'_i) \,, \tag{2.64}$$

where

$$\beta_i = -\frac{1}{2T_H R_i^2} \frac{m}{M(M-m)} n_i^2 \,, \tag{2.65}$$

$$\beta'_i = -\frac{2R_i^2}{T_H} \frac{m}{M(M-m)} \omega_i^2 \,, \tag{2.66}$$

$$\nu_i = \frac{1}{2\pi i T_H R_i^2} \frac{n_i}{M-m} \,, \tag{2.67}$$

$$\nu'_i = \frac{2R_i^2}{\pi i T_H} \frac{\omega_i}{M-m} \,, \tag{2.68}$$

$$\tau_i = \frac{i}{2\pi T_H R_i^2} \frac{M}{m(M-m)} \,, \tag{2.69}$$

$$\tau'_i = \frac{2iR_i^2}{\pi T_H} \frac{M}{m(M-m)} \,. \tag{2.70}$$

In general, as we argued above, the contribution from the KK-modes and windings could be just a constant and the dependence on the momentum k could be encoded in

$$\frac{\alpha^2-1}{\alpha^2} \frac{\beta^2-1}{\beta^2} \left(\frac{mM_2}{4}\right)^2 \exp\left(-\left(\frac{\alpha}{\sqrt{\alpha^2-1}} + \frac{\beta}{\sqrt{\beta^2-1}}\right) \frac{k^2}{2T_H} \frac{M}{2mM_2}\right) . \tag{2.71}$$

Note that if we measure the energy of the one of the decay string with mass m, we find a temperature:

$$T_m = \frac{T_H}{2} \left(\frac{\alpha}{\sqrt{\alpha^2-1}} + \frac{\beta}{\sqrt{\beta^2-1}}\right)^{-1} \left(1 - \frac{m}{M}\right) . \tag{2.72}$$

This reflects both the recoil effect and the influence of the KK-modes and windings.

It is remarkable that when $\alpha = 1$ or $\beta = 1$, the decay is completely suppressed. This interesting limit corresponds to the BPS condition $M = Q_-$ or $M = Q_+$ in the superstring case. As we show in the next section, all the discussion here could be applied to the superstring. Therefore, the fact that the BPS string states are stable is reflect in the above special limit. Actually, the suppression not only happens in the decay to the final massive strings, it also happens in the thermal radiation of the massless particles [12]. Moreover, from (2.71), the decay of the near BPS string states are greatly suppressed.

The measure of the integral in Γ^k is $dN_2 \sim M_2 dt$, where we define

$$t = \left(\frac{\alpha}{\sqrt{\alpha^2 - 1}} + \frac{\beta}{\sqrt{\beta^2 - 1}}\right)\frac{k^2}{2T_H}\frac{M}{2mM_2}\,. \tag{2.73}$$

Expressed in an integral over t

$$\Gamma^k \approx const.A'_{d_c} g_c^2 (2\pi T_H)^{-D+1}\frac{M}{m} m_R^{-\frac{D-1+d_c}{2}} \int_0^\infty t^{\frac{D-3-d_c}{2}} e^{-\frac{t}{T_H}}\, dt\,, \tag{2.74}$$

where $m_R = \frac{m(M-m)}{M}$, and $A'_{d_c} = \prod_{i=1}^{d_c} R_i^{-1}$. Thus including the phase space factor, we have a Maxwell-Boltzmann distribution for t, which could be taken as a measure of the total kinetic energy. Also the mean kinetic energy released per decay could be characterized by

$$\langle t \rangle = \frac{D - 1 - d_c}{2} T_H\,, \tag{2.75}$$

which is independent of the masses and satisfies the equipartition principle in $D - 1 - d_c$ noncompact spatial dimensions. Carrying out the integral, which is just a Γ-function, we get

$$\Gamma^k \approx A'_D g_c^2 \frac{M}{m} m_R^{-\frac{D-1+d_c}{2}}\,, \tag{2.76}$$

with $A'_D = \text{const.}\ A'_{d_c} T_H^{-\frac{(D-1+d_c)}{2}}$.

Then, we take into account the fact that there are $\rho(m)dm$ levels between mass m and $m + dm$, with $\rho(m) = \frac{1}{4}m$, so the total emission rate of strings of mass m is

$$\frac{d\Gamma(m)}{dm} = \Gamma_m(M, n_i, w_i)\rho(m) = \frac{d\Gamma^k(m)}{dm} \sum_{(n_{1i},n_{2i})} \mathcal{G}^{KK} \sum_{(w_{1i},w_{2i})} \mathcal{G}^w\,, \tag{2.77}$$

where

$$\frac{d\Gamma^k(m)}{dm} = \Gamma^k \rho(m) = A''_D g_c^2 M m_R^{-\frac{D-1+d_c}{2}}\,, \tag{2.78}$$

with $A''_D = \frac{1}{4}A'_D$. It can easily be checked that if we set $d_c = 0$, the above result reduces to the flat space case considered in [14] as expected. However, here is a puzzle. Naively, from semiclassical point of view, one may expect to interpret the probability for two points with distance m on the string to meet is inversely proportional to the volume $m^{D-1-d_c/2}$, where $m^{1/2}$ is the mean distance between two points and $D - 1 - d_c$ is the noncompact spatial dimensions. In other words, the less the noncompact spatial dimensions, the more possible the two points on the string to meet, inducing the breaking of the string. But (2.78) tells that this is not true: it seems that the more we compact the dimensions, the less possible is the string break.

3. Superstring decay: flat space

In this section, we calculate the decay rate of a typical massive type-II string state in the 10-d dimensional Minkowski space. We consider first the emission of NS-NS states, which we will argue later is the dominant decay channel. The basic quantity to compute is one piece of the semi-inclusive amplitudes squared as defined in (2.7) and (2.8),

$$F = \frac{1}{\mathcal{G}(N)} \sum_{\Phi_i|_N} \sum_{\Phi_f|_{N'}} |\langle \Phi_f | V(k) | \Phi_i \rangle|^2. \tag{3.1}$$

Carry out the projection

$$F = \frac{1}{\mathcal{G}(\mathcal{N})} \oint_C \frac{dz'}{z'} z'^{-N'} \oint_{C'} \frac{dz}{z} z^{-N} \operatorname{Tr}\left[\frac{1+e^{i\pi F}}{2} V^\dagger(k,1) \frac{1+e^{i\pi F}}{2} z'^{\hat{N}} V(k,1) z^{\hat{N}}\right], \tag{3.2}$$

where F is the world sheet fermion number.

We will mostly follow the procedure of [14], and the main task is to check whether there is still a recursion relation when the fermions are included.

Convert the above tree level trace to a cylinder correlator

$$\operatorname{Tr}\left[\frac{1+e^{i\pi F}}{2} V^\dagger(k,1) \frac{1+e^{i\pi F}}{2} z'^{\hat{N}} V(k,1) z^{\hat{N}}\right] = f_{NS}(w) \left\langle V'^\dagger(k,1) \frac{1+e^{i\pi F}}{2} V'(k,v) \right\rangle, \tag{3.3}$$

where

$$f_{NS}(w) = \sum_{n=0}^{\infty} \mathcal{G}_{NS} w^n = \frac{1}{\sqrt{w}} \operatorname{Tr}\left[\frac{1+e^{i\pi F}}{2} w^N\right], \tag{3.4}$$

with $\mathcal{G}_{NS}$ the state density of the NS sector. Note that now the massless level already involves 1/2 unit of excitation, so the factor $1/\sqrt{w}$ should be included.

We define $\mathcal{I}_n$ in the same way as in (2.18). On the cylinder we use vertex operators in the 0 picture in the notation of [18]. For example, the massless state is

$$V_0 = (\xi.\partial X + i\xi.\psi k.\psi) e^{ik.X}. \tag{3.5}$$

The state generated by $\psi^\mu_{-\frac{3}{2}}$ has the vertex operator

$$V_1 = (\xi.\partial^2 X + i\xi.\partial\psi k.\psi) e^{ik.X}. \tag{3.6}$$

More highly excited states can be constructed with higher derivatives and more terms with building blocks as above.

These facts suggest that the correlator in (3.3) can be factorized as in (2.24), imposing the same gauge conditions. Now the polynomial is also a function of the fermion correlators and their derivatives

$$\langle V'^\dagger(k,1) V'(k,v)\rangle = \hat{\psi}^{m^2} \mathcal{P}(\Omega, \partial_\tau \Omega \ldots; \Lambda, \partial_\tau \Lambda, \ldots), \tag{3.7}$$

where we define

$$\Lambda(v,w) = \langle \psi(1)\psi(v)\rangle. \tag{3.8}$$

Actually there are two subtleties in arriving (3.7). One is that though the vertex operator of the massive string state might be just the linear combinations of the terms which are the product of chiral and anti-chiral parts, the contribution from the contraction of two exponentials is universal and the end result could still be written as (3.7). The other subtlety is the gauge choice which allow the contraction between the exponential and the terms without exponential vanishing. As in bosonic case (2.20), such a gauge is believed to exist in the critical dimension.

What is new for the superstring case is that due to the presence of fermions, the correlator in (3.3) is a summation over spin structures. There are four spin structures on the cylinder. Restoring spin structures, the above correlator or Green function can be written as

$$G_{(a,b)}(w_1-w_2)=\langle\psi(w_1)\psi(w_2)\rangle_{(a,b)}\,, \tag{3.9}$$

with $a,b=0,\frac{1}{2}$ labelling the periodicity

$$G_{(a,b)}(z+2\pi)=e^{2i\pi a}G_{(a,b)}(z)\,,\qquad G_{(a,b)}(z+it)=e^{2i\pi b}G_{(a,b)}(z)\,. \tag{3.10}$$

The Green functions for the so-called even spin structures can be explicitly written as

$$\begin{aligned}
\langle\psi^\mu(w_1)\psi^\nu(w_2)\rangle_{(0,\frac{1}{2})} &= \frac{\alpha'}{4\pi}g^{\mu\nu}\frac{\vartheta_2(\frac{w_1-w_2}{2\pi}|it)\vartheta_1'(0|it)}{\vartheta_1(\frac{w_1-w_2}{2\pi}|it)\vartheta_2(0|it)}\,, &(3.11)\\
\langle\psi^\mu(w_1)\psi^\nu(w_2)\rangle_{(\frac{1}{2},\frac{1}{2})} &= \frac{\alpha'}{4\pi}g^{\mu\nu}\frac{\vartheta_3(\frac{w_1-w_2}{2\pi}|it)\vartheta_1'(0|it)}{\vartheta_1(\frac{w_1-w_2}{2\pi}|it)\vartheta_3(0|it)}\,, &(3.12)\\
\langle\psi^\mu(w_1)\psi^\nu(w_2)\rangle_{(\frac{1}{2}.0)} &= \frac{\alpha'}{4\pi}g^{\mu\nu}\frac{\vartheta_4(\frac{w_1-w_2}{2\pi}|it)\vartheta_1'(0|it)}{\vartheta_1(\frac{w_1-w_2}{2\pi}|it)\vartheta_4(0|it)}\,, &(3.13)
\end{aligned}$$

We can explicitly check their periodicity by using properties of ϑ functions

$$\begin{aligned}
\vartheta_1(\nu+1|\tau) &= -\vartheta_(\nu|\tau)\,, &(3.14)\\
\vartheta_2(\nu+1|\tau) &= -\vartheta_2(\nu|\tau)\,, &(3.15)\\
\vartheta_3(\nu+1|\tau) &= \vartheta_3(\nu|\tau)\,, &(3.16)\\
\vartheta_4(\nu+1|\tau) &= \vartheta_4(\nu|\tau)\,. &(3.17)
\end{aligned}$$

These three spin structures are permuted under modular transformations, so to ensure modular invariance they should always be kept at the same time.

For the doubly periodic case ($a=0,b=0$), which is often called odd spin structure, the existence of zero modes of the Dirac operator make things subtle(see for example [18]). It is modular invariant itself and does not mix with the other three under modular transformation. And it is well known that this sector does not contribute to the correlation functions if the insertion of the vertex operator cannot eat the zero modes, while in our consideration of very massive strings, the corresponding vertex operators can swallow the zero modes without trouble.

Note that in our convention, the arguments in the $\vartheta(\nu|\tau)$ function read

$$\nu=\frac{\ln v}{\ln w}\,,\qquad \tau=-\frac{2\pi i}{\ln w}\,. \tag{3.18}$$

To get a second contour we make the transformation $v' = w^{-1}v$, which amounts to replace $\vartheta(\nu|\tau)$ by $\vartheta(\nu - 1|\tau)$, or $G_{(a,b)}(z)$ by $G_{(a,b)}(z - 2\pi)$.

Using (3.10) we get

$$\Lambda(w^{-1}v, w) = \pm\Lambda(v, w)\,. \tag{3.19}$$

The minus sign in (3.19) is irrelevant since we always have even numbers of fermions in our vertex operators after GSO projection.

Taking derivatives of ϑ functions we can see that derivatives of Λ get at most an extra irrelevant minus sign under the transformation $v' = w^{-1}v, w' = w$. So it turns out that the fermion correlators have the same good property as ∂X correlators that they (effectively) do not change under our transformation. This is after all not a surprise since we know, for example from bosonization, that two fermions correspond to the derivative of a bosonic field, and GSO projection enables us to always have paired world-sheet fermions.

The contour integrations on contour C'_v and around the singularity $v = 1$ both carry on to the superstring case. The latter integration depends only on the asymptotic behavior of the correlators of e^{ikX}, ∂X^μ and $\psi\psi$ near $v = 1$, while the last two have the same order dependence on τ

$$\langle \partial X(1)\partial X(v)\rangle \sim -\frac{1}{\tau^2} \sim \langle \psi\psi(1)\psi\psi(v)\rangle\,. \tag{3.20}$$

With proper normalization we get the same result as in the bosonic case.

Thus we get the same recursion relation for $\mathcal{I}_n$ (2.35) as in the bosonic string [14]; the only change is to replace space-time dimension 26 by 10.

Using the expression for NS state density

$$\mathcal{G}_{NS}(N) = \oint \frac{dw}{w} w^{-N} f_{NS}\,, \tag{3.21}$$

we get the superstring semi-inclusive decay rate as a series summation

$$F = \sum_{p=1}^{A} \left(n - m^2\left(p - \frac{1}{2}\right)\right)\mathcal{G}_{NS}\left[N - np + \frac{1}{2}m^2(p^2 - p)\right] + F_{NU}\,, \tag{3.22}$$

with N the mass level of the initial string; $n = N - N'$, N' the mass level of one of the final strings states; m the mass of the other of the final strings whose state is fixed from the start. The above expression has the same form as in the bosonic string.

Note that we considered only the NS sector, and in the following we will argue in the spirit of [13] that NS emission is the dominant decay channel for N large, and R sector is suppressed by $1/N$, with N being the initial string mass level. Here for the semi-inclusive amplitude squared (3.1), it is more convenient to introduce only a projector for one of the final states

$$F = \sum_{\Phi_i|_N} \oint \frac{dz'}{z'} z'^{N-N'} \langle \Phi_i | V(k,1)^\dagger \frac{1 + e^{i\pi F}}{2} V(k,v) |\Phi_i\rangle\,. \tag{3.23}$$

Before GSO projection, the lowest states in R sector are already massless. Their vertex operators are of the form [18]

$$V^R(u^\alpha; k^\mu) = e^{-\frac{\phi}{2}} \bar{u}^\alpha \Theta_\alpha e^{ikX}\,, \tag{3.24}$$

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where the ϕ term comes from superconformal ghosts, and the Θ term, called spin fields, comes from worldsheet fermion zero modes. Compare this with the massless state (3.5) in NS sector, we see that to get to the same mass level, NS sector needs one more piece of excitation of the weight of ∂X than for R sector. The same argument works also for the higher mass levels.

Relative to R sector, this extra excitation will contribute in (3.23) a factor of order $\langle \Phi_i | \partial X(1) \partial X(v) | \Phi_i \rangle$ for each term in NS sector. Ignoring dependence on the worldsheet coordinates, $\partial X \partial X$ acts mainly as the energy-momentum tensor which can be seen by noting that the X scalar contribution to the energy-momentum tensor is

$$T^X(z) = -\frac{1}{\alpha'} \partial X^\mu \partial X_\mu \,. \tag{3.25}$$

So $\langle \Phi_i | \partial X(1) \partial X(v) | \Phi_i \rangle$ is of order the mass square of the initial string state which scales as N with N large. This extra factor of order N makes NS sector the dominant decay channel. And our truncation to NS sector in the above is seen to be enough at the leading order.

4. Superstring decay with compactification and KKLMMT scenario

We can also compactify type-II strings on the torus. According to the above calculations, we will obtain the same formulas as in (2.37), (2.47)–(2.50). Now the state density has the asymptotic form

$$\mathcal{G}(N_L) \approx 2^{-\frac{13}{4}} N_L^{-\frac{11}{4}} e^{\pi\sqrt{8N_L}} \,. \tag{4.1}$$

The computation of the multiplicative constant in the *r.h.s.* is included in the appendix A for completeness.

The results on the toroidal compactifications can be used to estimate the massive string decay rates in the more realistic KKLMMT model [11], where the type-IIB strings live in a highly warped throat with Klebanov-Strassler geometry [19]. It was shown in [20] how to use the string reconnection probability for toroidal compactification to study corresponding processes in the warped geometry. We will make use of their treatment.

A general warped geometry

$$ds^2 = H^{-\frac{1}{2}}(Y) \eta_{\mu\nu} dX^\mu dX^\nu + H^{\frac{1}{2}}(Y) g_{ij} dY^i dY^j \tag{4.2}$$

will induce a potential [20]

$$V(Y) = \frac{1}{2\pi\alpha' H^{\frac{1}{2}}(Y)} \tag{4.3}$$

on the string worldsheet, this potential confines the string to a small region in the compact dimension near the position where this potential has a minimum. Fluctuations in these compact dimensions [20]

$$\langle Y^i Y^i \rangle = \frac{\alpha'}{2} \ln \left[1 + \frac{1}{2\pi\alpha'^2 H^{\frac{1}{2}}(0) \partial_i^2 V(0)} \right], \qquad \text{(no sum on i)} \tag{4.4}$$

provide an effective compactification volume for strings living in. When $\partial_i^2 V$ is small in string units, that is, the geometry varies slowly on the string scale, we can look upon the strings in such a warped geometry as in a box with the effective compactification volume, and flat space calculations above applies.

Near the base of Klebanov-strassler solution [19], the geometry is locally $R^3 \times S^3$ with the warp factor

$$H(Y) = H(0)\left(1 - \frac{b' r^2}{g_s J \alpha'}\right) \tag{4.5}$$

near the origin of R^3. $H(0)^{-\frac{1}{4}}$ is determined to be of order 10^{-4} [20], and b' nearly 1 [21]. And J counts the number of flux.

As in [20], write the minimum volume of a six-torus as $V_{\min} = (4\pi^2\alpha')^3$, and the effective volume of the six transverse dimensions as $V_\perp$. The dimensionless quantity $\frac{V_{\min}}{V_\perp}$ can be used to measure the effects of compactification. Quantum fluctuations on the R^3 contribute $\frac{(2\pi)^{\frac{3}{2}}}{\ln^{\frac{3}{2}}(1+g_s J)}$ to $\frac{V_{\min}}{V_\perp}$ [20]. For S^3, quantum fluctuations give $\frac{(2\pi)^{\frac{3}{2}}}{\ln^{\frac{3}{2}}(H^{\frac{1}{2}}(0))}$, and its volume constrains the factor to be no less than $\frac{4\pi}{(g_s J)^{\frac{3}{2}}}$ [20]. So finally we get

$$\frac{V_{\min}}{V_\perp} \approx \inf\left[\frac{4\pi}{(g_s J)^{\frac{3}{2}}} \frac{(2\pi)^{\frac{3}{2}}}{\ln^{\frac{3}{2}}(1+g_s J)}, \frac{(2\pi)^3}{\ln^{\frac{3}{2}}(H^{\frac{1}{2}}(0)) \ln^{\frac{3}{2}}(1+g_s J)}\right]. \tag{4.6}$$

Here we need not to worry about winding modes, and as a rough estimate we can also neglect the contribution from KK modes. The flat metric total decay rate (2.48) for a typical string of mass M to two strings with mass m and M_2, when these two modes are excluded, can be simplified

$$\Gamma[M \to m + M_2] \approx A_{D-d_c} \frac{1}{V_\perp} \frac{g_c^2}{M^2} \mathcal{N}^2 \mathcal{G}^2 k^{D-3-d_c}, \tag{4.7}$$

with $\mathcal{N} = \mathcal{N}_L = \mathcal{N}_R$, and $\mathcal{G} = \mathcal{G}_L = \mathcal{G}_R$, and compactification volume $V_\perp = \prod_i^{d_c} R_i$. Remember that the initial string is averaged, and the final two strings both sum over given mass level. For the warped geometry, (4.7) represents results seen by a 10-dimensional observer. And the quantities measured by the 4-dimensional observer, in our case the mass and the momentum, get red-shifted. Thus we should multiply M, m, M_2 and k all by a red-shift factor $H^{\frac{1}{4}}(0)$. Now the mass shell conditions read like

$$\alpha' M^2 H^{\frac{1}{2}}(0) = 4(N-1). \tag{4.8}$$

Let's look at the eq. (2.74). Now we need to replace t by $H^{\frac{1}{4}}(0)t$ and m_R by $H^{\frac{1}{4}}(0)m_R$ to get Γ^k replaced by $\Gamma^k H^{-\frac{15}{8}}(0)$, with $D = 10$ and $d_c = 6$ explicitly inserted.

Then the emission rate of a string with mass m from a typical string of mass M in the Klebanov-Strassler geometry can finally be estimated to be

$$\frac{d\Gamma_{KS}(m)}{dm} \approx A_{KS} g_c^2 H^{-\frac{13}{8}}(0) M m_R^{-\frac{15}{2}}, \tag{4.9}$$

with $A_{KS} = \frac{1}{16\pi^5}\frac{V_{\min}}{V_\perp}$.

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With $H^{-\frac{1}{4}}(0) \sim 10^{-4}$, we see that the emission rate is suppressed by a factor of order 10^{-26} in this warped geometry! With A_{KS} no larger than 10^{-4}, g_S no larger than unity, the multiplicative constant in (4.9) is no larger than 10^{-30}. Thus in this warped geometry, the decay channels for a massive string to two massive strings are greatly suppressed.

We note that the situation is different if one of the two final strings is massless. It is found [13] that the decay rate is proportional to the mass of the initial string. So including the redshift factor, it can be written as

$$\Gamma_{\text{massless}} \sim g_c^2 M H^{\frac{1}{4}}(0)\,, \tag{4.10}$$

from which we see that this decay channel is enhanced by order 10^4. Thus in such warped geometries, massive strings may lose their energy mainly through gravitational radiation.

5. Discussions

We calculated the average decay rate of a typical massive string state in superstring theory, generalizing the earlier results obtained in the bosonic string theory. We did this calculation with a torus compactification, which at the first look is not realistic enough. However, we see that the calculation can be applied to the brane-inflation model of KKLMMT, we find that the decay is greatly suppressed. We believe that this shall have important phenomenological implications.

To apply our calculation to the problems concerning cosmic strings, we need to know how fundamental strings are generated, for instance in the process of brane-anti-brane annihilation. With that knowledge, we can write down a set of differential equations on the distribution function of massive strings, making use of our result. We may need to take the evolution of the universe into account too. The approach outlined here may replace the usual simulation process to predict the number and the distribution of massive cosmic strings that remain to this day in the sky.

Particularly, it has been argued in [22] that gravitational radiation is not an efficient enough energy loss mechanism for the string network to be consistent with present observation. Other mechanisms are needed to prevent the network from quickly dominate the universe. And from our calculations above, we can see that the large suppression factor is a generic feature for string interactions in the warped geometry with three massive strings involved. Thus it is important to study more carefully the energy loss mechanisms for F-D string networks in warped geometries.

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A. Compute the state density for superstrings.

The asymptotic densities of the states for bosonic and superstrings are standard textbook knowledge (see for example [17] and [18]). However, in our computation we need to know the detailed numerical coefficients for superstrings (Manes has done for bosonic string in [14]), which are often not explicitly demonstrated, so we include this appendix to write them out. We follow the whole procedure of [17].

Consider first open strings and NS sector only. The state degeneracy $\mathcal{G}_{NS}(n)$ is given by

$$f_{NS}(w) = \mathrm{Tr}\frac{1+e^{i\pi F}}{2}w^N = \sum_{n=0}^{\infty}\mathcal{G}_{NS}(n)w^n 8\prod_{n=1}^{\infty}(\frac{1+w^n}{1-w^n})^8\,, \tag{A.1}$$

with N the summation of the bosonic and fermionic number operators. Generalization of Hardy-Ramanujan formula gives

$$\prod_{n=1}^{\infty}(\frac{1+w^n}{1-w^n})^{-1} = \vartheta_4(0|w) = (-\frac{\ln w}{\pi})^{-\frac{1}{2}}\vartheta_2(0|e^{\frac{\pi}{\ln w}})\,, \tag{A.2}$$

where the modular transformation of ϑ function

$$\vartheta_4(0|\tau) = (-i\tau)^{-\frac{1}{2}}\vartheta_2(0|-\frac{1}{\tau}) \tag{A.3}$$

has been used, with

$$\tau = -\frac{i\ln w}{\pi}\,. \tag{A.4}$$

As $w \to 1$, the second argument of ϑ_2, which now reads

$$\tau' = -\frac{1}{\tau} = -\frac{i\pi}{\ln w}\,, \tag{A.5}$$

approaches ∞. We know from the expansion

$$\vartheta_2(0|\tau') = \sum_{n=-\infty}^{\infty} e^{i\pi(n-\frac{1}{2})^2\tau'} \tag{A.6}$$

that

$$\vartheta_2(0|\tau' \to \infty) \to 2e^{\frac{i\pi}{4}\tau'} = 2e^{\frac{\pi^2}{4\ln w}}\,. \tag{A.7}$$

Thus (A.2) is asymptotically

$$\prod_{n=1}^{\infty}\left(\frac{1+w^n}{1-w^n}\right)^{-1} \to \left(-\frac{\ln w}{\pi}\right)^{-\frac{1}{2}} 2\exp\left(\frac{\pi^2}{4\ln w}\right). \tag{A.8}$$

From (A.1), the state degeneracy $\mathcal{G}_{NS}(n)$ can be expressed as a contour integral on a small circle around $w=0$

$$\mathcal{G}_{NS}(n) = \frac{1}{2\pi i}\oint \frac{f_{NS}}{w^{n+1}}dw\,. \tag{A.9}$$

To compute the above integration, we make a saddle point approximation near $w = 1$. The power of w can be put on the exponential

$$\mathcal{G}_{NS}(n) = \frac{1}{2\pi i}\oint 8\left(-\frac{\ln w}{\pi}\right)^4 2^{-8}\exp\left[-\frac{2\pi^2}{\ln w} - (n+1)\ln w\right]dw\,, \tag{A.10}$$

to get the saddle point at

$$\ln w_0 = \frac{\sqrt{2}\pi}{\sqrt{n+1}}\,, \tag{A.11}$$

where expansion can be made

$$\ln w = \ln w_0 + iu\,. \tag{A.12}$$

Then $\mathcal{G}_{NS}(n)$ is approximately

$$\mathcal{G}_{NS}(n) \sim \frac{1}{2\pi}\frac{1}{32}\left(\frac{\sqrt{2}}{\sqrt{n}}\right)^4 e^{\pi\sqrt{8n}}\int_{-\infty}^{\infty}\exp\left(-\frac{\sqrt{2}n^{\frac{3}{2}}}{\pi}u^2\right)du\,. \tag{A.13}$$

Carrying out the integration over u we find

$$\mathcal{G}_{NS}(n) \sim 2^{-\frac{13}{4}}n^{-\frac{11}{4}}e^{\pi\sqrt{8n}}\,. \tag{A.14}$$

Or using $n \sim \alpha' m^2$, write it out in terms of mass

$$\mathcal{G}_{NS}(m) \sim 2^{-\frac{13}{4}}\alpha'^{-\frac{11}{4}}m^{-\frac{11}{2}}e^{\pi\sqrt{8\alpha'}m}\,. \tag{A.15}$$

Here we use the convention $\mathcal{G}_{NS}(m)dn = \mathcal{G}_{NS}(n)dn$, different from [17].

At this point, we also note that R sector has the same expression. And combine the left and right pieces together we arrive at the expression for closed strings

$$\mathcal{G}^{cl}(n) = [\mathcal{G}^{op}(n)]^2 \sim 2^{-\frac{9}{2}}n^{-\frac{11}{2}}e^{4\pi\sqrt{2n}}\,. \tag{A.16}$$

Taking care of the difference between the mass shell conditions of open and closed strings ($\alpha' m^2 \sim 4n$ for closed strings), the state degeneracy for closed string as a function of mass reads

$$\mathcal{G}^{cl}(m) \sim 2^{\frac{13}{2}}\alpha'^{-\frac{11}{2}}m^{-11}e^{\pi\sqrt{8\alpha'}m}\,. \tag{A.17}$$

Thus open and closed strings have the same Hagedorn temperature

$$T_H = \frac{1}{\pi\sqrt{8\alpha'}}\,. \tag{A.18}$$

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